Einstein v. Bergson on time

Bergson's philosophy could be summarised in four words: time is not space.

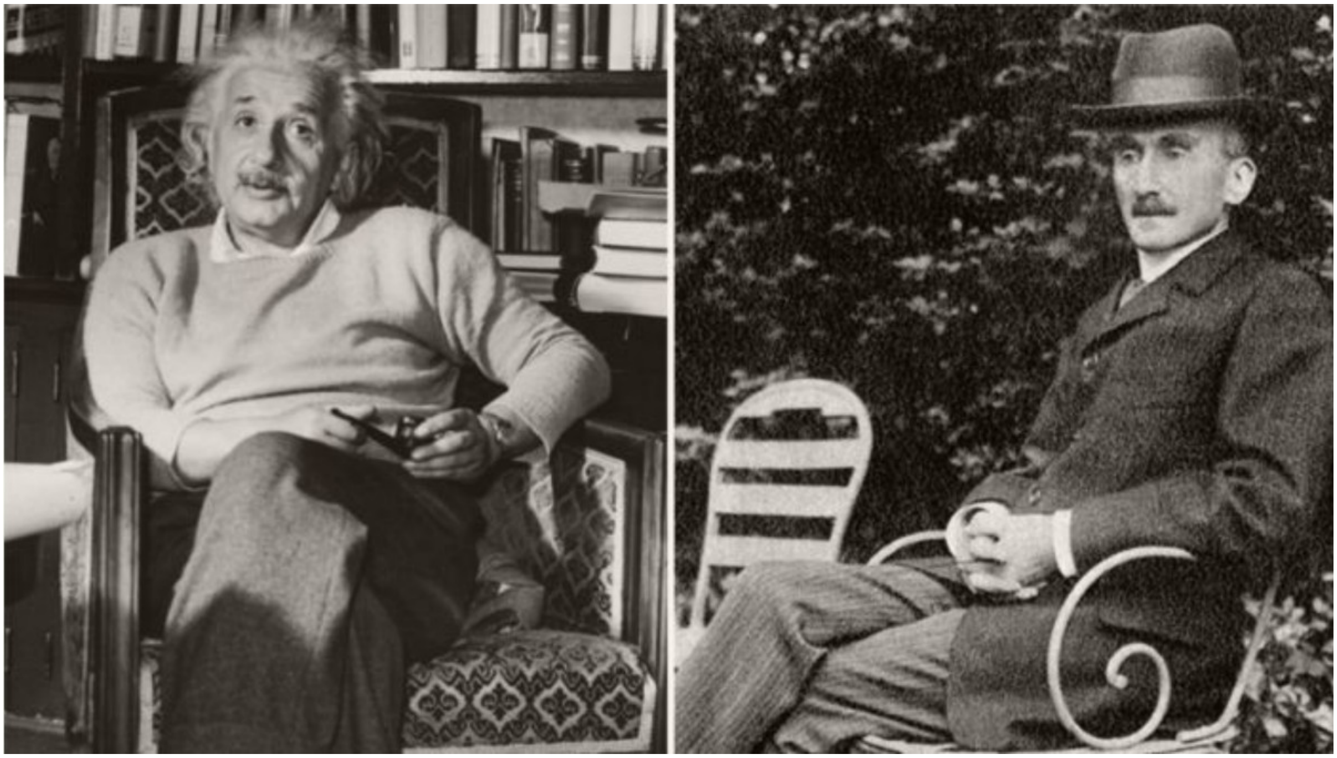
– Emily Herring

For people like us who believe in physics, the separation between past, present and future has only the importance of an admittedly tenacious illusion.

– Albert Einstein[1]

Everything has a history. And history happens in time. And if time doesn't exist, there is no "room," so to speak, for history. And hence no room for anything else... Darwin is incompatible with Einstein.

– Bianco Luno



Albert Einstein and Henri Bergson - Getty Images

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[1] From a letter of condolence on the death of physicist friend Michele Besso to his family. The full quote reads, "Now he has again preceded me a little in parting from this strange world. This has no importance. For people like us who believe in physics, the separation between past, present and future has only the importance of an admittedly tenacious illusion." Source.

For most of the 20th Century and even now, in the opinion of many, Albert Einstein was the most influential thinker on the subject of time. He published his first work on relativity with implications for the concept in 1905. But the importance of his theory of relativity was not acknowledged for the first quarter of that century largely because Henri Bergson, a philosopher,[2] disagreed with him on the implications of relativity for *human* concerns. In fact, Einstein did not get the Nobel Prize for his work on relativity because of Bergson's qualms. Instead, Einstein got the prize 1921 for work he considered less important.[3]

On 6 April 1922, Bergson and Einstein met and publicly confronted each other… Einstein won out in the ensuing public relations war between the sciences and the humanities which continues to this day. Many people, even philosophers, know very little about Bergson and what they have heard has been filtered by famous critics such as Bertrand Russell. But Bergson's ideas remain subversively pervasive and threaten a comeback. During his lifetime and since, Bergson's account of temporal experience has appealed to women and feminist thinkers as well as existentialists, pragmatists, and "creatives" everywhere.

It is difficult to exaggerate the intellectual rock star Bergson was during this period. How many philosophers today can fill a venue of almost 2,000 seats with an audience? Or had ever done so before him? None, that I know of. There were riots outside his lectures. Famous contemporary philosophers, John Dewey and William James sung his praises. Mathematicians and physicists, Henri Poincaré and Hendrik Lorentz aligned with Bergson against Einstein. Bergson was cited among the greatest thinkers of all time, along with Plato, Descartes, Hume and Kant.[4] His understanding of time as *duration*, at the heart of his thought, was considered revolutionary. Duration is what it *feels* like to experience time.

But Einstein's equations were not about feelings. They were presented as describing reality as it is apart from any experience we have – or even *could* have. Where is the *relevance* in that?

...

Last time we looked at theories of time, developed mostly by analytic philosophers, and how they relate – *or don't* – to ordinary experience. Experimental philosophers have recently remarked that the ordinary person's experience of time does not make much of the metaphysical implications of these theories – the more physics-based, the less so. Again, behind the scenes, we can discern notions traceable to Bergson.

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[2] Bergson was also trained as a mathematician and considered becoming a mathematician before deciding on philosophy. He was among the few philosophers at the time competent to understand Einstein's work in detail. His book *Duration and Simultaneity* (1922) was a close reading, both appreciative and critical, of Einstein's work on relativity. Einstein read this book, and while disagreeing with Bergson's interpretation of time, privately (at least) acknowledged that Bergson grasped what was a stake. See Canales' various accounts in Resources below.

[3] "[F]or his services to Theoretical Physics, and especially for his discovery of the law of the photoelectric effect." Source.

[4] To say nothing of his influence on the arts. "...[O]utside science, in normative realms such as ethics and aesthetics, the irrelevance of the 'objective' parameters we have been assessing [are] par for the course. The literary world, for example, has long been comfortable with conceptions of time that are tightly at one with phenomenal experience. Consider *stream of consciousness* in the work of modernist fiction writers, for instance: Gertrude Stein, Virginia Wolf, James Joyce, Marcel Proust, William Faulkner, Clarice Lispector, and many others... including philosophers who inspired them, such as William James and Henri Bergson. None of the so-called 'defining' characteristics we have been discussing – dynamism, direction, order, and a multiplicity of time instants – are taken seriously. Time *may* have a 'feel' to it, or several such, or none stable enough to be a 'defining' characteristic…" from our earlier discussion of McTaggartian theories of time.

This time we will look at the *other* story of time. Misunderstanding still pervades the debate. How much descriptively innocent, how much normatively motivated? How much stemming from genuine alternative understandings of *the medium of experience*[5] and how much motivated by more local and contingent egotistic and political partialities? It is tempting to resolve the conflict now by saying they were *not* talking about *the same thing* – Einstein and Bergson. But at the time there was enough passion and emotion behind the debate between their two camps to assure us that they, at least, thought differently.

Objectivity and correction

Again, earlier, we focused on those theories of time that more or less attempt to characterize "objective" time. The older theories from this set were broadly empirical, the newer ones increasingly physics-based. Physics, the most rigorous and abstract of the sciences, does not usually advertise itself as a specifically "human" physics but as a mathematical description of the world – and mathematics certainly not as "our" mathematics – not *anthropocentric*, *peculiar-to-us,* or *local* mathematics – but mathematics, pure and simple, as though this way of apprehension offered the only portal on reality, and, thus, the final arbiter of right-thinking about it. Any way that clashes with this way of apprehending time is liable to be called "psychological" or "illusory" in a normative sense, indicating a deprecated way of thinking that it would behoove us to retreat from.

Early, historically "objective," theories begin with widely held perceptions or intuitions[6] that time involves events undergoing motion or change, direction or orientation, and *at least* reflect *some* order or structure about the universe or world *external* to our perspective or even our existence. More recently, increasingly sophisticated theories of physics seem bent on progressively whittling away at these traditionally attributed properties of time – a kind of conceptual striptease where the concept of time is slowly disrobed to reveal more of what is essential about it, on the one hand, and what was all along a cultural or contingent accretion, and so gratuitous, on the other. Considerations of conceptual and theoretical economy, Occam's Razor-thinking,[7] dictates the trend. There is a growing realization that the equations of physics have no need to ascribe properties such as dynamism, direction, order – or even multiplicity[8] to time to preserve their mathematical integrity. So why should we – going about doing non-physics – bother with these old notions? The striptease seems to reveal "time" for what it is: at best, a side effect or epiphenomenon of other theories (e.g., gravity); at worst, nothing at all. Hence, dispensable.[9]

Supposing these developments move us closer to the truth...

The normative element in suggesting these notions should be abandoned as "naive" and "outdated" calls for more elaboration. Normativity, the *shouldness* of a judgment, comes *from* somewhere, from a place of need or desire. Let's not forget that. So long as science confines itself to describing the world accessible to it in a take-it-or-leave-it way, normativity (or value imposition)[10] is not involved. The world at the edge of our senses (aided or unaided) is indifferent to us and all of our designs. Science *must* presume this. Otherwise, it is dabbling in religion or a belief system of a sort.[11]

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[5] Is it space or time or a contrived "space-time"?

[6] But, apparently, *loosely* held. See the X-phi results cited in our earlier writeup.

[7] "Plurality should not be posited without necessity" source or "that, done with less, done with more, is done in vain."

[8] More than even *one instant* is *physically* superfluous.

[9] As with the story of "ether": we were eventually given to understand light propagated itself just fine without needing to assume any such thing. Psychology, etc., will have to catch up with physics.

It may be true that humans possess no other epistemic entry mechanism for grasping information than by forming beliefs, but facts presumably are what they are independently of whether anyone believes them.[12] So the intended *objects of science* – stable, non-subjective judgments about the world – do not require the kind of doxastic investment we can get hot under the collar about. Suppose a double-blind randomized and controlled trial, over a large and representative sample size, confounders accounted for, etc... produces unambiguous results about a new medical treatment, *so what?* The proper *scientific* attitude should be: if tomorrow another trial produces different results, then correct your steering and proceed, fully prepared and expecting to have to correct it *again* very soon... and so on. *When the correction stops, the science stops.* Does the science stop when it runs up against a bedrock of “truth”? A claim that it does – or could – necessarily invites suspicion; it would evince manifestly unscientific thinking on its face, the stale odor of a “belief system” in the air.[13]

If we can’t always explore so matter-of-factly, it indicates something about *us*, the *urgency* of the investigation *for us* – *not* the objects of scientific investigation. Truth, assuming there is any, has all the time in the world at its disposal to wait us out. We don’t.

This is a point Bergson drives home again and again in this philosophical account of time. Einstein, in contrast, seems either confused or impatient in his pronouncements on the concept of time.

The illusoriness of time

> *How did a paper about clocks,[14] and only certain kinds of clocks,*
> *become a paper about time in general?*
>
> – Jimena Canales

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[10] Beyond the sparest form of it implied in the inductive logic or rational thinking requisite to begin to think scientifically. Why even this form of normativity is acceptable also requires a story, but not here.

[11] The suggestion that science *only* presumes evolutionarily-backed survival instincts, and nothing else, is incoherent: evolution is a description of a process, not a justification for it. Species, no less than individuals, go extinct. *So what?* You *must* be presuming something over and above a “process” to be disturbed by the flippancy of this reaction. Specifically, you *must* presume there is a “should” attached to survival... We need to do better than this to argue against the antinatalist who sees nothing of the sort.

[12] Assuming a form of realism for the nonce. Einstein (at least at this point in his career) expressed himself as a realist.

[13] Science is essentially inductive. It takes its cues from past experiences. It can only *rationally* conclude on the basis of these. If the past has taken a certain course, our *rational* expectations of the future cannot ignore this course. Deductive logic permits other possibilities. But this purer logic is indifferent to the past.

[14] Referring to Einstein’s 1905 paper on special relativity, whose claim was still quite modest: measurements derived from certain types of clocks were unreliable. The question is from Canales’ book *The Physicist and the Philosopher: Einstein, Bergson, and the Debate That Changed Our Understanding of Time*, *p. 91.*

Einstein said “time is an illusion”: in the letter of condolence, at the pivotal conference on 6 April 1922 before some of the leading intellectuals of the time, and in many other contexts. It wasn’t a gaffe. So there are questions about what he meant by the claim:

1. Was he *joking*? Was he being figurative, humorous, or intentionally provocative?

2. Or did he mean that some *specialized understanding* of time was illusory? – as in: “if by ‘time’ you mean this… then you are not referring to anything real.”

3. Or did he mean it *literally?* – as in: the “time” people (in their capacities as “ordinary people”) mean whenever they talk about time – *that* time is *not* real – with all the implications normally associated with calling something “illusory.”

Rhetoric

On the rhetorical interpretation of Einstein’s remark, he had reason to be provocative at the conference where he was invited to speak on the 6th of April 1922. Seventeen years earlier he had published his theory of special relativity with radical implications for time. The theory did not make the splash he expected it would or felt it deserved. If he was right, what he had to say was revolutionary, but it seems few people were getting it. He and everyone around him knew well that the *then* reigning conception of time was that of Bergson. So when Einstein said *il n’y a donc pas de temps des philosophes* (“the philosopher’s notion of time does not exist”) in front of an august assembly of leading intellectuals, everyone within earshot understood Einstein meant to target the view of time associated with Bergson – who was in attendance. Bergson’s popularity was, in part, explained by his philosophical corroboration of the significance of *felt* time – time as experienced by humans going about their business. His view of time resonated like few others ever had. Challenging Bergsonian time would finally drive home to those still not taking Einstein seriously that the physicist’s insights on time demanded attention. If he meant to shock, Einstein succeeded. By all accounts, he had felt unappreciated and saw the occasion of the conference to fix that. This might suggest that being literal was not as important to him as raising eyebrows.

A term of art

On the specialized, mincing interpretation, Einstein didn’t really mean what it seems he did. His French, after all, was not the best and may have precluded more nuanced expression. This is the sober take: if Einstein’s calculations were correct, whatever role the concept of time played in physical descriptions of the world, it was not the popular one. The time that mattered to physics was *mensurable* time – time that could be parsed out using *spatial* metaphors derived from concrete *spatial* entities such as clocks, pendulum swings, timelines, etc. Physics has always been more at home with *space*. Time, by contrast, appears sometimes as a helpful bystanding – but never central – role. The useful, and thus relevant, conception of time is that which relates one *spatial* event to another: the hour of your doctor’s appointment picked out by *the hands of a clock* (a physical configuration in space) with your *physical presence* in the doctor’s office: the *simultaneity* of these events. “You made it on time, what else matters?" Time as a tool for *coordinating* physical/spatial events is sometimes useful.[15] But this understanding of time is only grasped via *spatial* terms. If time has any non-illusory significance, this is what it is. To have a useful ontology is to have fixed or mathematically predictable coordinates. Time, absent *spatial* events to coordinate, lacks these.

Regarding the notion of time just described, if that's what Einstein intended, then it is what Bergson called "spatialized time." If *this* understanding of time, i.e., "clock time," seems illusory and insubstantial to the point of being mathematically superfluous, then one can scarcely wonder that we may grow skeptical of its amounting to anything. If Einstein meant that the significance of time *in mathematical descriptions of the universe* is gratuitous, Bergson was inclined to agree, or at least he had no objection.[16] Bergson was early on enthusiastic about the *physics* of Einstein's view of time.[17] Thus, there is room to construe their views as compatible. Bergson and Einstein were *not* talking about the same thing. What Einstein denied was not what mattered to Bergson. Once confusion around this cleared, the drama would dissipate.

But that's not what happened.

Einstein would go on to repeat regularly the remark about time's unreality without qualifications,[18] never acknowledging that there was any confusion to resolve by making distinctions, even when others hinted there might be. Moreover, he repeated the claim in contexts that clearly included Bergsonian "human time" in the dismissal – right up to near the end of his own life, as in the letter of condolence to the survivors of his predeceased friend and physicist Michele Besso.

***Literally*, there is no such thing as time**

We are left to conclude that Einstein was *not* being rhetorical, *nor* did he intend to merely split hairs (as philosophers are wont to do), rather, he insisted there is *only one coherent understanding* of time, and *that* is the one physics describes, and, in *that* understanding, time's reality is highly problematic to the point of justifying the claim that it is illusory. Full stop. He was being *literal*.

Assuming he meant the claim literally, a lot follows, perhaps more than he bargained for.

To subjectivists, pragmatists, intuitives, creatives and anyone busily mired in everyday experience – to all those *not* doing physics, including many physicists themselves when they are not doing it, Einstein's claim, not surprisingly, was arresting. Bergson thought the claim was plainly false on the normal understanding of what time is. And, it seems, it is *this* normal understanding of time that Einstein had in mind. Hence, the clash.

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[15] And if you are constantly checking the time – constantly obsessed with "clock time," you are arguably not functionally living, rather, busy *sampling* your life.

[16] As one may say, from the perspective of interstellar space, there is no up or down, left or right, except relative to the speaker. Nothing objective corresponds to these notions. Likewise, nothing corresponds to time in physics. "Objective" means here: *not* relative to any perspective. Time *not relative to anything* is nothing. (We leave aside discussion of *absolute* time, since neither Bergson, nor Einstein, were defenders of that. Both were time relativists. Bergson to human experience; Einstein to the integrity of mathematical descriptions of reality.)

[17] Again, as a trained mathematician, Bergson was positioned to scrutinize Einstein's theory with more comprehension than most non-physicists, something even Einstein *privately* admitted in his journal, while saying the opposite *publicly*.

[18] The number of physicists even today who in one way or another have inherited Einstein's suspicions about time is legion. Spatialized time expositions crowd Youtube, such as those of physicists and science-explainers: Neil de Grasse Tyson, Bill Nye "the Science Guy," Sabine Hossenfelder, Carlo Rovelli… *et al*, not to exclude Stephen Hawking.

The problem goes deep. It is not just non-scientists, doing "culture," who have problems with the claim, but *nearly all the sciences except theoretical physics* (and perhaps parts of chemistry) are implicated in Einstein's charge of dealing in unrealities. Empirical (as opposed to theoretical) science is caught up in the charge of illegitimacy. Isn't there a *sequence* to experimental procedure? Aren't experiments *first* designed around an hypothesis, *then* implemented, *then* data is gathered and *only then* results assessed? The "thens" mean nothing?

Take evolutionary and developmental biology (and, by implication, all sciences with temporal presuppositions downstream of these: psychology, sociology, anthropology, economics, history, etc.)… How are they supposed to work if time is *only* a "tenacious," psychological proclivity, with no basis in reality?

If Einstein was right, was Darwin wrong?

Pure unadulterated science and subjective experience

The color blue, from a physics point of view, is about an electromagnetic wavelength between 470 and 495 nanometers. That's *all* it is from a physics point of view. If you mean anything but that when you employ the concept of blueness to describe an experience, you may (charitably) be reacting to a psychological phenomenon peculiar to you, or your kind, when exposed to the wavelength. The qualitative aspect of the experience is an artifact of your sensory apparatus and the usefulness of this aspect, presumably, is itself a consequence of evolutionary advantage.[19]

We may concede as much. But it is a different claim to suggest that the *qualia* of blueness[20] is "illusory" or "unreal" or eligible to be relegated to the dustbin along with once popular beliefs such as that the sun circles the earth or the earth is flat or that a "p-value" *by itself* resulting from a randomized controlled trial is a sure fire indicator of anything significant, let alone conclusory, about the effect of a drug on a biological condition.[21] It is a different claim because it makes *no practical difference* to the wear patterns on the soles of my shoes whether the earth is spherical or not. *No difference*, which circles which, sun or earth, to my calendar or clocks. Perhaps it should, but, in fact, it makes *little difference* to the pharmaceuticals available to me if p-values go the way of… the h-index,[22] when much larger psycho-economic, perspicuously evident, factors *chiefly* determine the selection of *authorized* chemical treatments.[23]

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[19] Imagine a visual artist adopting the physics point of view in regard to color – except ironically, playfully. Where does *irony* fit in the periodic table? Not next to iron.

[20] What it is like to experience blue.

[21] The "p-value" of a scientific finding is a commonly used measure of the weight or credibility it should be given. It is a statistical calculation of the likelihood that a given finding results from chance and not from an suggestive correlation or a causative event. P-values ranges between .0 and 1. The closer the p-value is to 1 the more likely the result is coincidental; the smaller the decimal fraction, the less likely the result was chance or the more likely there is a meaningful connection between hypothesis and result. A good exposition of the concept is here. However, p-values as measures of statistical significance are being challenged: "Retire statistical significance," Valentin Amrhein, Sander Greenland, Blake McShane and more than 800 signatories call for an end to hyped claims and the dismissal of possibly crucial effects. *Nature*, 21 March 2019. We seem to want a single number to tell us what to think or do. *This is desire, not science.*

[22] A measure of the correlation between citation and scientific merit. "The h-index is no longer an effective correlate of scientific reputation," Vladlen Koltun and David Hafner, *Plos One*, 2021. Should it come as a surprise that scientific advancement is *not* about who you know or who knows you?

Spatialized time

Bergson believed the qualitative experience of time was the philosophically interesting aspect of the concept. In fact, time, the kind that matters to humans,[24] has no essential *quantitative* aspect. What your clock measures is an artifact of space and mathematics. Its undoubted utility is no reason to suppose it describes an essential feature of reality for beings decked out with the awareness we have. *Utility, as a measure of reality, is a mistake.*[25]

Bergson thought metaphysics was *not* in the business of being useful. An indication of utility is a sign that we are not attempting to *describe* but, rather, to *accomplish* something. Good, but such a qualification on "understanding" reduces it to a search for coping mechanisms. A phenomenologist may be content with such modesty. But realists, as Einstein was (at least at this point of his career), do not see themselves as partners in such an accommodating endeavor.

The mathematical description of the universe that defines the project of modern physics is a specialized[26] empirical activity. As with all sciences, physics proceeds via models and concepts to investigate percepts. These tools are ultimately rooted in metaphors and similes. We have no way to begin describing new or mysterious phenomenon except by setting already more familiar images beside them to see if these help make the new or mysterious more comprehensible. It is an accident of our constitution that we have a clearer grasp of space than we do of time. We are *visual*. We see things move or change. Things are observed to move in a direction. We make out order or disorder in the structure of things around us. The ideas of motion, direction, and order stem first and foremost from discernments in our visual field. These observations and inferences happen against a background we call "space." Space for us is the stage on which action and stable situatedness perform. It is the still background *against* which – or the container *in* which – things move or change from something to something, from more or less order. While space itself may not be an immediate percept like experiencing an apple on a table, we seem compelled to believe the apple has a *location* in space. This is a primal inference. What becomes of the apple begins *at* the location in space the apple has been perceptually assigned. Ditto for every object of observation or inference – up to and including the entire universe. Thus, the laws generalized from regularities in percepts that harden into the laws of physics are seen as comprehensive and exceptionless.

Since science – and physics, in particular – is an attempt to investigate the objects of the empirical universe, space (the anthroposcopically inferred idea) is the stage these objects inhabit. Hence, there is temptation to grasp all the furnishings of the universe as perturbations of – or “in” – space. We are predisposed to think we “understand” space. The rigorous mathematical description of these perturbations do not seem to require reference to anything distinct from space.

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[23] E.g., *lucrative* vaccines and antivirals

[24] Time's utility, when it has it, matters. But utility matters *for* something. For some human end.

[25] For present purposes, "reality" is a state of affairs *irrespective* of the existence – let alone, of the interests – of an observer. If the apprehension of the state is conditioned or colored by the capabilities or requirements of a class or a member of a class of observers, it is a *phenomenology*: either intersubjective or subjective. Again, Einstein was a realist in this regard: it is clear he did not think of himself as merely doing anthropocentric descriptions. He was not remarking on the view from his personal or species portal. With the aid of principles that he took to be defining of the *only* ultimate reality available to us – *physical* reality, he derived conclusions. That many others agreed – and agree – with him suggest the credibility of the principles is at least intersubjective. *But more than that?*

[26] "Spatialized," too.

And since we do not experience time as we do objects in space, nor are we as pressured to make stable inferences to time from experience, time’s grounding in reality, relative to space, may come to seem tenuous, provisional, even absent. Time’s flux or fluidity, its ebb and flow, may count against its reality. Time is different, Bergson insisted, from *any* object of interest to physics. Time is a *becoming,* never a *being* – always a *process,* never a *thing* – except as we are compelled to take sample slices of it for study. All the empirical sciences are confined by their method to this practice. Necessarily and usefully so. So long as we remain cognizant that *we are sampling, not fixing, reality.* Bergson thought Einstein lost sight of this.

Bergsonian “duration” is not sequential, not mensurable. The past, future, and *felt* present interpenetrate. We worry about the future *now*. We worry about what *in the future* will be referred to as now or past. In the past, we had experiences in *anticipation*. “Now” is a soupy amalgam of all this. Time may even temporarily vanish as when we are absorbed in a task or thought. Time then ceases even to be remarked as “now.” It has no dimensions, can expand or contract, can vanish entirely, can jumble its “contents.” What can physics make of this but wonder if it is “real”? But unless physics is *philosophically* taken to be the arbiter of reality, why is this even a concern? Einstein did, and many of his followers do, assert the *philosophical* authority to pronounce on “reality,” and that is the problem.

Feelings about time are often adequate where a similar imprecision about space more often can be disastrous. Space wants, sometimes demands, to be measured. We can be *late* to an appointment and the appointment may *still* happen. Get the precise *location* wrong and it won’t. Time is, relative to space, forgiving. Such slack is incompatible with the standard understanding of the laws of physics.

Music, Bergson remarked, is, among arts, indigenous to time. Attending to a single note in a musical passage as though, by itself, it was a constituent of the emotional impact of the piece[27] as a whole is an error if you think the relation is like that of a load-bearing brick in a wall. The forces acting on and through the brick can be precisely calculated and its structural and explanatory role in the structure as a whole thus exhausted. *Not so with a note or a chord.* The musical significance of a note depends not only on what notes come *before* and come *after* and which are *simultaneous,* but on what notes *don’t* but *might* have or might still come before or after or been simultaneous: i.e., the *predispositions* and *expectations* of the auditor. And these memories and expectations depend on what? The entire bio-, psycho-, socio-, historical and cultural configuration of the field against which the experience of the note is happening. Not just the person attending the note, not just their personal history and their current, past, and anticipated psychic states but that of the history and culture that conditions these and that of all person(s) whose auditory systems are sharing the experience; not only a discrete experience, not only the past, but the hopes, fears and anticipation of the individual and their cultural milieu. In short, the past and the future reaching back *and* forward indefinitely, unstably, and dimensionlessly in all the “directions” of experience.[28]

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[27] And consider a musical piece or a phrase *without* emotional impact? In what sense would it *still* be music?

[28] I like – or don’t – a piece of music that you do... Even if we both like it, this is for different reasons. The full exposure of those reasons would show us to have lived *distinct* lives. From a similar investigation, even within a piece of music (regarding a particular passage in the music) the same conclusion. All the way down to the note or chord. That we might choose to

Physics wants to describe reality in term of dimensions. Space is described in *three*. Time, to the extent it has any, has sometimes been conceded its own *fourth*.[29] But, again, space, the *more* grounded thing is being used as the basis for explaining the *less* grounded thing, time. A *grounded* thing is a thing that has GPS coordinates. Time does not have these. We may mark out clock ticks and time slices but these, again, are *spatial* metaphors. *Discrete* description is always spatial. This is the problem with scientific attempts to say something interesting about time. Such attempts are always using the tools of one medium to describe "in" another. Already, the use of "in" should raise suspicion.

Bergson thought conceiving of time in terms of spatial notions a mistake. It is a lame *after*thought. It construes time as derivative of, or an optional appendage or concession to, our spatial contingency. The core idea, as far as physics is concerned, is space. It has geometry. Time has nothing like that. The objective theories of time, we discussed earlier, sought, at least initially, to graft onto the ostensibly better understood concept of space an explanation for the experience of time. With Einstein, this became explicit. Post-Einstein, the grafting has increasingly come to seem labored. It explains nothing. *Neither for the physicist, nor for the person on the street.* Ergo, time, *that of the physicists*, one might be excused for thinking, does not exist.

Bergson: time ≠ space / Einstein: space-time

But time is not space, Bergson insisted.

Time has nothing to do with space. It is not a dimension of some other thing nor does it have "dimensions" or properties in any sense derived from those space is defined by. To think that it does is to be carried away by the assumption that what is more obvious can always enlighten us about the less obvious, that the visual field and all it contents and the inferences it favors offer useful or accurate similes for understanding the less immediate, less *sensorily* definable idea of time.

Time takes time.

The experience of it happens in its own medium. It does *not* take up space. Nor is it governed by regularities native to the space of concern to physics. Time is not "in" anything. *That* is a *spatial* preposition. Time has no need to be conceived as being "in" anything. If it is true that only things that can be "in" something can possibly have an address or location in the universe suitable to being called "real," then time may fail to exist. If so, it is indeed disqualified from existence. That is, *if* having a location and dimensions (spacial notions) are requisite for possessing reality.

But why accept time and space are related in this way? One may suppose this because of a longstanding investment in taking space as primary. Much of modern physics is so invested.

It appears Albert Einstein was so invested (as Newton had been before). He relegated time to being an adjunct of space, i.e., "space-time." (Note: he didn't say "time-space.") But physics since Einstein has

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focus on aspects of consensus (both of us may find the chord *meaninglessly* dissonant or *irredeemably* jarring), that, too, is for different reasons stemming from different histories and expectations in time. The durations each of us settle on as explanatory have respectively different "dimensions" and their aspects different saliences.

[29] Dimensions beyond these push the subject even further from relevance.

increasingly come to realize the superfluity[30] of tacking on time to space. Little wonder. Einstein might have done better to just drop the reference to time since nothing outside the specialized project of physics would remark the absence of this *spatialized* notion of time, a lame vestige of an ill-conceived idea to start with. His willingness to concede time an *adjunct* status suggests he was not quite ready to dismiss it altogether. Its psychological "tenaciousness" still exerted friction for him against going that far. Nevertheless, his gesture in this direction paved the way for his successors to go the "distance" and explicitly do the dismissal thing.

Hence, Einstein was able to make sense of consoling himself[31] and the relatives of deceased physicist and long-time friend Michele Besso – his supposed fellow "believer" (his word) in physics – that nothing much is lost when you die since death purportedly happens *after* life and there in no meaningful *before* or *after* in physics. Temporal predicates, along with their medium: time, are illusory. "Tenacious illusions" aside, they cannot refer. Get over it, Einstein was suggesting. *It would save us a lot grief (so to speak) if psychology could catch up with physics.*

Einstein seemed to think that, if it is reality we are concerned with, then the usual way we talk about time should be purged of meaningful notions of *past*, *present*, and *future*, as well as *before*, *now*, and *after*. He also described the propensity to take time seriously as "tenacious," acknowledging that it is hard to give up thinking of time in the way that we have been accustomed to. The difficulty, however, he says explicitly, is *psychological*, and, he seemed to think, psychology is easier to overcome than altering the laws of physical *reality* – which describe the only reality there is. Is that so, really?[32]

Time and consciousness

If we are stubbornly inclined to *feel* otherwise, the reluctance to face physical reality (it cannot be emphasized enough is the *only* true reality there is) is an artifact of our psychology – which, in turn, is a consequence of our evolutionary history in a *necessarily* (in Einstein's terms), *purely* spatial environment. And as for *evolution*, itself? A purely *spatial* thing? – since what could evolution mean *atemporally*? Was Einstein obviating Darwin? Evolution happens *exclusively* in space among creatures with stubborn illusions that they have or had or may still have a history? Illusory natural selections? Really? What sense can we make of this? Can we imagine Darwin theorizing in a temporal vacuum? If Einstein was right, does Darwin's theory become unintelligible?

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[30] The superfluity is corroborated by the trend noted in our last discussion involving McTaggartian conceptions of time. From every direction, it seems, from x-phi folk explorations, from formal McTaggartian and purely physics-based formulations of it, from the Bergsonian duration conception, time conceived *spatially* is losing its grip on reality.

[31] Assuming Einstein was not trying to be funny. Was Epicurus joking? – "So death, the most terrifying of ills, is nothing to us, since so long as we exist, death is not with us; but when death comes, then we do not exist. It does not then concern either the living or the dead, since for the former it is not, and the latter are no more." (Epicurus, "Letter to Menoeceus.") To the extent death is an "experience," when we are alive we do not have it – and when dead we don't have experiences of any sort. Shall we conclude we are immortal since death can never be "a thing" for us? "Loss," likewise, for Einstein cannot be real. Supposedly, gain cannot be either. Not only is it a "tenacious illusion" that we die, but *that we ever lived* to the extent either of these conditions occur in time. When we say something is an illusion, the implication is that it would be a good idea to abandon belief in it and anything that depends on the belief. Einstein's attitude seems to accord with this presumption.

[32] There is a similar suspect assumption often made regarding the relative malleability of "nurture" versus "nature." Quite apart from whether these concepts can be coherently separated, where do we get the notion that one is *obviously* more tractable and amenable to reform than the other? Are *social* constructions really *more* subject to reconstruction than *biological* givens? This is a question separate form the desirability of *any* reconstruction.

At best, there is conceptual room for Darwin's being a correct theory about an *illusory collective* experience. Illusions matter. Put another way: Darwin was a phenomenologist(?)

This might work conceptually if we banish from all the contents of psychology[33] the significance that labeling phenomenal experience "illusory" entails.

If time is not real, nothing that "lives" *in* – or depends *on* (to persist with spatial "picturing") – time is *either*: not history, not any notions of mental development over time, not consciousness, nor any of its sometimes purported contents. Evolutionary theories and much else, *out the window!* There is no meaningful *location* for these things to happen. There is space, with or without the "-time" appendage (a principle of parsimony would prefer *without*) and the rules that supposedly govern non-temporal activities in it, but no theories to be conscious of, *because* no consciousness, *because* no medium for consciousness to be "contained in." (To keep up the spatialized talk.)

For Bergson, *the experience of time is tantamount to consciousness.*

Consciousness comes in irregular parcels. We experience these as durations, he suggests. So long as we remain forever cognizant of our spatial terms (e.g., "parcels") as flexible, as metaphors start out being, and leery of their hardening into concepts with fixed "dimensions" from which scientific constructs – models, laws, theories, etc. – emerge, then scientific description may proceed coherently to do its thing – trying to make sense of the world: because creatures like us have vested interests in protracting our survival. Science has this human – *all-too-human* – limitation. This limitation stands in the way of its objectivity. Science wants to be useful. Does it want to describe reality *as it is?* Leave that to the philosophers. Philosophy is *not* a gainful occupation, nor can it properly pretend to be, Bergson said as much. *If it threatens to become useful, it threatens to become science.* That may be a victory for *some* human interests, maybe most, but it robs realist pretensions of any but rhetorical force.

Einstein overreached when he pronounced on the reality of time, saying it was an illusion. In venturing to say something *useless*, he dabbled in philosophy. That was his mistake. And a host of his intellectual descendants inherited it.

For a span of something we call "time," we are a locus of conscious experience. This span Bergson preferred to call "duration." It does not have fixed dimensions. Long or short, it may narrow to a moment or dilate to include a life time, perhaps beyond that even. Most often, though, something more sustainably manageable in between.

If the duration of conscious experience is infused with imaginary "subjective" things, these are at least *our* things. And, if they never rise above the imaginary, neither do we.

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[33] Not only everything conceptually downstream of psychology: sociology, anthropology, history, etc., but somethings upstream as well: evolutionary and developmental biology. Perhaps *some* aspects of physical chemistry escape, those with some hope of being reduced to physics.

Einstein's theory, time paradoxes, and other sciences

The historical evidence suggests that Einstein had not initially thought through the *biological* implications of his theory. Intimations of these consequences were likely sparked by the extrapolations worked out by his physicist friend and early promoter Paul Langevin, whose fanciful thought experiment, now known as the "Twin Paradox" has continued to fire the imagination both of the public and many (otherwise, sober) physicists to this day.

The conclusion Langevin drew from his "Twin Paradox" thought experiment is that due to the peculiarities of travel at great speeds (dramatically so at near light speed) if one of a set of twins remains on earth while the other travels at great speed from the earth to a distant star and then returns, the traveling twin will arrive younger than the one who remains. Time dilates, slows down, at speed.[34]

However, this picture seems to take earth as *the* frame of reference for deciding *which* twin moved. Why not take up the traveling twin's frame of reference? If so, then it is the earth-bound twin who moves away from the traveling twin at great speed and, hence, is the one who ages more slowly. Which is it? They can't both age less than the other? Or can they? Suppose we interpret the picture as one in which both age more slowly, resulting in no aging difference at all *relative to each other*. They both stay younger longer together. From what possible frame of reference would this outcome be noticed? In other words, the units of measure would *also* dilate or contract as the perspective of the measurement changed. Differences from any possible perspective would cancel each other out at the point of being measured.

So is it *meaningful* to say time dilates at speed?

The problem is that, as a consequence of Einstein's theory, there is no *absolute* frame of reference we, as observers, may take up in order to judge.

There have been attempts to resolve the paradox by appealing to differences in acceleration between the two: the traveling twin had to have experienced acceleration (and deceleration) while the earth-bound one didn't. But, again, this *assumes* a frame of reference from which we can say acceleration happened, but there is no obvious reason to say *who* is accelerating from *whom*. Acceleration, too, is relative.

There have been other attempts to remove the paradoxical aspect but none that have achieved consensus. The issue, theoretical or practical, about relativity is purely a problem of physics and to be settled, if possible, not by philosophers. The most help a philosopher may offer is a reminder that the terms in contention need precise and stable definitions.

But the point of *philosophical* contention between Einstein and his major critics at the time, Bergson, foremost among them, was not that there is no absolute time. To draw the further conclusion that time is an illusion, *in addition to being relative*, we need to assume that before Einstein's revelation, the consensus was that being real meant the same as being absolute. That was indeed an assumption made by many but not all. Bergson's conception of time, for one, would remain undisturbed by the result that time – *clock time*, initially, what it appeared Einstein meant – might be relative. On Bergson's notion, to say that time is relative is an understatement. Time on his view was *universally* relative: relative to quite literally everything. Einstein's was the more modest claim that time was relative to the point in space where its measurement is taken.

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[34] The twins were dubbed "Peter and Paul" by Bergson. He called them phantasms. Fiction had overtaken science in their conception. So their paradoxical character should not surprise.

The crux of the dispute, then, was the claim that time was *illusory*. That is different from saying it is *relative*.

Supposing time *unreal*, or a superfluous, unsubstantial notion, in any supposed *literal* sense, then it will have global effects. Langevin suggested his Twin thought experiment implied that a *biological* process (aging) could be slowed or sped along or even nearly arrested by relative speed differences. (Einstein himself didn't, at least initially, pursue these biological, "extra-physical" implications. Langevin got him started thinking about consequences for other scientific portals on reality such as the biological.) If we can imagine the slow aging of an *individual* organism by its speeding through space, even to the point of nearly stopping the process at near-light speed relative to a frame of reference *other* than that this moving organism experiences, then the evolution of entire species or ecosystems or any temporal processes, too, could be slowed or nearly stopped. *All temporal processes become illusory.* Since we are on a relativistic roll, why stop at biological individuals aging?

Perhaps our species evolution and/or individual development has *already* nearly stopped from a frame of reference vastly slower than ours. Perhaps with respect to *some* frame of reference ours has, for all our finite intents and purposes, stopped. As from another, we have long since become *extinct*. From the perspective of some universe traveling at near the speed of light relative to ours, *evolution* has nearly stopped.

It has been suggested that we may salvage the significance of time for evolutionary biology *while retaining the literal sense of Einstein's claim of the illusoriness of time* by asserting that though *cosmic* time may by unreal, *local* time (time within a single "light cone" where all the relevant elements are in such proximity that dilation effects are negligible, e.g., terrestrial environs) may still remain substantially meaningful and insulated from the sting of being labeled "illusory." But, if so, then a similar case can be made in defense psychology (etc.) and the phenomenal, subjective duration of Bergson. Indeed, if "local" is the saving grace, it is even more applicable to Bergsonian than to evolutionary time. Without a better explanation, the time of Darwin and Bergson stand or fall together. Einstein unambiguously dismissed the latter, how could he avoid doing the same for the former?

Moreover, relative to a *logically* possible world traveling *faster* than the speed of light, evolution is reversed. We came first, *then* the dinosaurs[35] – Darwin on the right track but going the wrong way? Certainly, current theories of physics do not permit anything, other than light, traveling even *at* the speed of light, let alone *faster*, but, we should remember: the theory of relativity is still a "theory," not yet promoted to a law of nature, *let alone of a law of thought*. Grant it time to mature: why so much pessimism about the inevitability of theoretical obsolescence in physics (or science, generally)? Where is the empirical evidence for the wisdom of such short-shortsightedness? Science is still young… patience. Tomorrow is a new day...

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35. You might have died and lived before you were born.

I said *logically*. In theory, there is always logical space for a lot of theory-obsolescing in the offing. *When the correction stops, the science stops.* It is only human to want to believe otherwise. We are mortal, our interests finite. We don't have all the time in the universe to do science with the patience *its method* requires. We dabble in – not *do* – science.

Resources

1. "This Philosopher Helped Ensure There Was No Nobel for Relativity: Henri Bergson's debate with Albert Einstein reached and swayed the 1921 Nobel committee," Jimena Canales, *Nautilus*, April 21, 2016. Or watch historian of science Canales' video presentation. The full story is described in her book, *The Physicist and the Philosopher: Einstein, Bergson, and the Debate That Changed Our Understanding of Time*.

2. Canales gives an extensive description in her articles, book, and video presentations, but here is the account, in French, of what happened on 6 April 1922 between Bergson and Einstein at the philosophical conference in Paris where Einstein presented his views on time and confronted his critics, includes relevant commentary by other prominent thinkers, the whole indicating the thinking at the time about time and where Einstein stood in relation to it. (*Centenaire de la société française de philosophie*, Les grandes conférences du XXe siècle: textes téléchargeables en intégralité.)

3. "Was Einstein wrong?" Adam Frank, *NPR*. A popular article on the Einstein/Bergson confrontation.

4. "Henri Bergson, celebrity: Women loved Bergson's philosophy of creativity, change and freedom, but their enthusiasm fuelled a backlash against him," Emily Herring, *Aeon*, May 2019.

5. The best entry into Bergson's philosophy is his own highly readable and succinct *Introduction to Metaphysics*: full text and free audiobook. Freely accessible digital versions of Bergson's main texts can be found online (or I can provide them on request).

6. An excellent detailed video exposition of Bergson is this playlist by Absurd Being (Nathan Hohipuha).

7. "How to Understand the Universe When You're Stuck Inside of It," Amanda Gefter, *QuantaMagazine*, 2019. "Lee Smolin has a radical idea for how to understand an object with no exterior: Imagine it built bit-by-bit from relationships between events." Though Smolin draws direct inspiration from Leibniz, not Bergson, the resonances with Bergson are striking. Not surprisingly, Leibniz was a major influence on Bergson, specifically, on time. Time is *not* space, Bergson and Smolin agree. Space is a physical *construction* of the relations among events, and events are creatures of time. Between time and space, if either is privileged or more fundamental, a case is made that it is time. See Smolin's talk on "Time Reborn," (also the title of his book). He leads the backlash against the "time in not real" camp in physics.

8. "Time consciousness: the missing link in theories of consciousness," Lachlan Kent and Marc Wittmann, *Neuroscience of Consciousness*, 2021, 7(2). The article argues that a Bergsonian-like insistence on the phenomenology of time is indispensable to progress in understanding consciousness. (Thanks to Mike Armstrong for the reference.)

Notes

Time questions

1. How does science relate to reality?
2. What is the relation of science to subjectivity v. objectivity?
3. Assuming a clear distinction between the subjective and objective views on "reality," what is their *relative* value?
4. What about the internal coherence of science when one branch, physics, denies the reality of others, evolutionary and developmental biology, in particular, (but all sciences downstream) by denying the reality of the essential medium in which they operate: *time*?

Postscript

from Bianco Luno, *Leçons en ténèbres,*

On science:

> The problem with doing science is that we can't handle the world *just as it is*, as the pig it presents itself as. We have to embellish it, put lipstick on it. Colorize it. Every pig needs lipstick. *Even in science!* – a consequence of the fact that we *must communicate* our observations and are bounded by the limitations of our biological, psychological, and sociological envelope place on communication. Science is not *supposed* to be about *us*, our vulnerabilities, tastes, or wishes. Rather about the world out there, the way it is or would be if we weren't here to observe and apprehend it. *That* world has no color as we understand it. Or much of anything else as we understand it. It certainly doesn't care. Curiosity about things that *do not care* is as good a definition as any of science. Things that are not good, nor evil, nor beautiful, nor ugly. That are unimaginably sterile – not to say, indecent, even pornographic – if only those terms were not themselves loaded with normative grime. This thought is surprisingly hard to grasp.

On time:

> Time not real? Yes, it is. Everything *else* isn't. Ask an insomniac. You must be asleep. (Luno's gloss on a remark of E. M. Cioran.)

Extended writeup for the topic hosted at

The Philosophy Club in March 2022

– Victor Muñoz

Oaxaca / Seattle

Appendix:

Time, infinity, calculus, and a theory of why mathematics education often fails

In *An Introduction to Metaphysics*, Bergson alluded to a problem with conceptualizing the concepts of limits and infinity in mathematics. Do math teachers often fail their students by not taking into account Bergsonian "human time" while introducing calculus? One mathematics pedagogy expert seems to thing so.

...

Math teachers sometimes use thought experiments like the following by way of giving students an intuitive grasp of how calculus works:

To calculate the area under a curve, parcel the area into discrete rectangles whose areas can be calculated straightforwardly. Those areas left out of the first parceling *event* are further parceled into rectangles and their areas calculated, constituting a second parceling *event*. The increasingly smaller areas not yet parceled already are further parceled and so on... The areas calculated as you proceed are summed. As you add smaller areas to the total the resulting number *approaches* the actual area under the curve.

Approaching the correct number is not the same as *achieving* the correct number. One can arrive at the *exact* area, for example, of a unit square with one foot (or inch, meter, light year, etc.) sides. Or by multiplying the lengths of twos side of a rectangle. One does not *approach* the correct area of a square or a rectangle. Assuming your measures for the sides are correct, one gets there *exactly*.

The method described by the math teacher for determining the area under a curve involves ever refined approximations. Thus, the student may understandably conclude that calculus does not give us exact results – *ever*. It is never 100% accurate.

But a mathematician will disagree.

At this point, math teachers often make a mistake, according to Colin Foster, an expert in mathematics pedagogy. The mistake helps explain why so many otherwise enthusiastic, capable students lose interest in pursuing mathematics beyond calculus.

The method of calculus is *not* about approximating your way to a result. It gives *exact* results *properly* understood. So we might ask: does the picture involving summing up areas of generated rectangles give us to understand calculus *properly*? Or is the picture misleading because it is poorly designed if the goal is a *proper* understanding? Explaining calculus this way must be seen as a provisional heuristic. It conveys an *idea* of what is going on, not a perspicuous explanation. A Wittgensteinian ladder: not intended to be more than provisional – a rough and ready introduction. Like a map is to the terrain it wants to elucidate.

For a proper understanding, however, a totally new concept must be grasped: that of a limit and infinity – concepts that have no features in common with the numbers that mathematics has involved up to this point in the student's education.

The problem is that, as the thought experiment is described, it implies *events in time:* successive events consisting of calculations in *time*. Such events consume time. Human time is finite. So is the intelligible time in which our machines operate. Therefore, in practice someone or some thing calculating in the exact way described in the experiment will *never* in a lifetime achieve the exact result of the area under the curve. Not in a million lifetimes. Worse: even after a gazillion lifetimes of calculations, the resulting number will *still* be an *infinite* distance away from the correct answer. Worse still: it will be the same distance from the correct answer as would be the case if the calculation had stopped after the very first calculation event(!), i.e., an *infinite* distance.

How is this *even* an "approximation"?

See the offense to logic? An "approximation" that is *infinitely* distant from exactness?

A student may be shown how to calculate the area of a unit square (a square with sides of 1 inch, 1 meter, 1 mile, 1 light year… whatever unit of measure desired). To quote Colin Foster, mathematics pedagogue,

> You can show students by shading in fractions of a unit square that ½ + ¼ + 1/8 + 1/16 + 1/32 +… is pretty close to 1, but for any finite number of terms there is always a tiny bit of the square left unshaded. We can make this bit of leftover square as small as we like by taking

> enough terms. However small you want to make it, I can tell you how many terms you need to take to make it smaller than that. But then students come away thinking that
>
> $\sum_{i=1}^{n} \frac{1}{2^i} < 1$,
>
> which, of course, is true for any *finite* n. The issue is what happens as $n \to \infty$. Do we want to say that when you "go to infinity" that somehow makes everything all right and that the partial sum then *equals* 1 exactly? Why should they believe this?
>
> Part of the problem here is in thinking of infinity as a process that happens in time – the "..." at the end is being taken to indicate that someone keeps on adding more and more terms. If you think of it like this, then you will never get all the way to 1, because the series will never be finished. But that's not what the notation means; it is referring to the *entire* infinite sum, as a complete finished object. [more context follows at the end of this commentary]

We can hardly blame the student for giving up on math after encountering calculus. The strain on intuition is quite severe. This can be because the thought experiment, as described, is misleading.

The picture that is offered of how calculus works neglects to introduce the concept of *infinity*. The reason mathematicians have perfect faith in the *method* of calculus (if not of this or that particular result) is because the number of iterative calculation events assumed by the method is infinite.

Plug in infinity as the "number" of times the calculations are repeated will give you the limit. The limit is the *theoretically exact* number corresponding to the area under the curve: do the calculations an infinite "number" of times (*not* to be confused with a gazillion times) and, yes, you will get the exact, 100% correct answer.

The student's mistake is to assume that infinity is a *large number*. The teacher's mistake is not to belabor the point that it is *not*. The teachers fails to stress how different the concept of infinity is from anything heretofore in the students mathematical education. And why might this mistake have arisen? Infinity is *not* a large number in anything like the sense of "larger than" used in describing 2 as *larger than* 1, or 10^{42} is *larger than* 17. Here is the critical point:

Infinity is not even a "number," let alone, a "large" one.

Infinity is *not quantitatively* larger than anything. It is a *qualitatively* different thing than any number we ordinarily conceive of as a "number." Adding infinity to the number 7 is like adding an apple or a square to 7. (Not one or two or three, or… apples. Just any instance(s) of the kind you *eat*. Not a square of certain dimensions but the *concept* of a square – a closed four-sided equilateral figure with interior angles of 90 degrees, whether the Platonic form or the outline of dice). *Without further explanation*, the operation is incoherent. It literally "does not compute." Addition is not defined for these addends.

Hence, the feeling of hocus pocus with infinite operations in calculus. There is a failure to dispel them.

The student's intuitions are right to be offended. The hocus-pocus is that ordinarily calculations in every facet of life from checkbook balancing… to moonshots *never* involve any person or any machine performing an *infinite* number of calculations. Always and everywhere calculations are done with proper numbers (i.e., *finite* ones) that relate to each other in *quantitative* ways, including being "larger than" or "smaller than." Always and everywhere these are *finite* calculations. Because *we* are finite and anything *we* are interested in *intelligibly* is finite.

Unless, of course, we are busy doing something theoretical. Unless we define a concept that may interact with proper numbers *without* being one of them. Such is the concept of infinity.

...

So, as Colin Foster, suggests, it would be less misleading to say that the method of calculus, while not practically exact, is indeed theoretically so. What the method delivers is a result to *whatever tolerance for inexactness or error you need or desire*. It gives a result "as close as you like" to whatever you want. The thought experiment was never, functionally, more than an analogy, a metaphor, if you like, for how humans might go about getting *useful but inexact* results. This point must be driven home to avoid disappointment.

The reason this requires special effort to drive home has very much to do with the fact that we are creatures of time – or *duration*, as Bergson would put it. Meaningful time, the kind that matters, is always finite and sometimes usefully measurable. Clock time, the experience of which, never fails to happen within duration; and thus never fails to be finite and open to measure. Sometimes the measure is meaningful as when clocks are important consults or data points for determining simultaneity (e.g., does the *physical* configuration on the clock face relate to my *physical* presence in the doctor's waiting room), sometimes irrelevant as when *felt* time is what matters ("time flies when you are having fun").

What are we up to? Whatever *that* is, we may, if we input careful measures, get useful results, calculus assures us. We can depend on getting results adequate to our needs or desires – assuming we don't conceive of our needs or desires as unbounded.

But note that *time*, as we normally understand it, figures nowhere in the *theory* of calculus.

Mathematics is not bounded by the fact that those who employ its methods are finite.

So why should *theoretical* physics? Physics that is *not* engineering? The relevance of time separates physics from engineering.

...

The idea of "infinite time," therefore, is an oxymoron. Like the notion of "married bachelor." Perhaps, it may help our theory of marriage to stipulate a non-folk definition for "married bachelor." But it will remain non-folk… until we convince the folk of its utility.

From “Is calculus exact?” by Colin Foster, (*Mathematics in School*, May 2018, The MA website www.m-a.org.uk) :

> Although I would avoid getting into epsilons and deltas and formal definitions at this level, I do try to use helpful language, like “arbitrarily close”, or “as close as you like to”, which I think is much better than “closer and closer 1 to”. For example, if $y = 1/x$ gets “closer and closer” to zero x as x tends to infinity, then it also gets “closer and closer to” any number *less than zero*. “Closer and closer to” doesn’t capture the idea of a limit. The important point is that y gets “as close as you like” to zero; i.e. if you decide how close to zero you would like y to get, provided you make x large enough you can always get even closer than that. The epsilon–delta definition of a limit is often presented as playing a game where, “You tell me how small you want the error to be, and I’ll tell you how close to the x value you have to be so that the error will always be smaller than that”. So, for our $y = x^2$ example, the challenge would be “You tell me how close to 6 you want the gradient to be, and I’ll tell you how close to 3 you have to make the x to achieve that”. The point is that no matter how small a difference from 6 you want to have, I can always get you closer than that by setting the x value sufficiently close to 3. Isn’t that good?
>
> But I find that this argument never really works very well with students. If they are thinking about the ideas and engaging in the argument, they always say something like, “I want to be *exactly* at 3. I don’t want a *small* error – I want no error at all!” If I say something like, “You can make your error as small as you like”, then they say “I’ll have it as zero then, please!”, and then I have to say, “Well, as small as you like, *but not actually zero*,” which seems to support them in the Stourton view that calculus is really very, very accurate but not totally 100% accurate. The takeaway message is that there’s always a tiny little error, and you can never completely get rid of it (but you can make it small enough to be insignificant for practical purposes).
>
> Actually, this problem is not specific to calculus; it is the same issue with any limit. You can show students by shading in fractions of a unit square that ½ + ¼ + 1/8 + 1/16 + 1/32 +… is pretty close to 1, but for any finite number of terms there is always a tiny bit of the square left unshaded. We can make this bit of leftover square as small as we like by taking enough terms. However small you want to make it, I can tell you how many terms you need to take to make it smaller than that. But then students come away thinking that
>
> $$\sum_{i=1}^{n} \frac{1}{2^i} < 1$$
>
> , which, of course, is true for any *finite* n. The issue is what happens as $n \to \infty$. Do we want to say that when you “go to infinity” that somehow makes everything all right and that the partial sum then *equals* 1 exactly? Why should they believe this?
>
> Part of the problem here is in thinking of infinity as a process that happens in time – the “...” at the end is being taken to indicate that someone keeps on adding more and more terms. If you think of it like this, then you will never get all the way to 1, because the series will never be fin-

ished. But that’s not what the notation means; it is referring to the *entire* infinite sum, as a complete finished object.

One way to think about the size of the gap is to say that if it is non-negative and we can make it smaller than any positive number that we care to name, then it must be zero (Note 3). We can make the gap smaller than any positive number, just by taking enough terms of the sequence. The student may ask us how many terms it would take to do this, and then what would we say – “infinity”? It’s tempting to say something like “It turns out that if you take infinitely many terms, then the gap becomes *exactly* zero”, with maybe a “You’ll prove it at university”, and that is either taken on trust or rejected as false. But thinking about the gap we can ask “Can you think of a non-negative number that is smaller than any positive number?” The answer must be zero, but of course, we had to sneak in “non-negative” there! It may seem a bit weird, because in epsilon–delta terms we are saying that we can make epsilon smaller than any positive number, so it can be zero, but we actually began by saying that epsilon had to be strictly greater than zero.

Perhaps this isn’t really something that we can demonstrate at any level, even at university…